

# Electron transport in barrier delta-doped coupled quantum well structures

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**Abstract** : In this invited talk, we briefly review the work done on low temperature electron subband mobility  $\mu_n$  in various  $\delta$ -doped systems. We then analyse  $\mu_n$  in double quantum well structures, which exhibits the tunneling coupling in addition to confinement effect. We discuss the importance of intersubband coupling on  $\mu_n$  in a multisubband system. The effect of screening of the scattering potentials by 2DEG on subband electron mobility is also highlighted.

**Keywords** : Electron mobility, delta doping, 2DEG, double quantum wells, GaAs/AlGaAs, GaAs/InGaAs systems.

**PACS No.** : 72.10.-d

## 1. Introduction

The study of properties of two dimensional electron gas (2DEG) in semiconductor structures has been a subject of interest from both technological as well as fundamental point of view [1–4]. The invention of modulation doped heterostructures (MDH) [5,6], which afforded a physical separation of the electrons from the ionized parent donors, led to a realization of an exceedingly high electron mobility  $\mu$  of the 2DEG [7]. A new challenge thus emerged to understand the mechanism of electron transport [8] in the modulation doped heterostructures and also to explore the possibility of other systems exhibiting such properties.

## 2. Delta-doped systems

In recent years, attempts have been made to realize 2DEG by confining the distribution of dopant atoms over a very narrow width, ideally one-monolayer called  $\delta$ -doping [9,10]. The confinement potential in the  $\delta$ -doped systems is simply formed by the interaction between ionized impurities in the doping layer and the delocalised electrons around it. This gives rise to quantum size effect and therefore subband structures [1]. The  $\delta$ -doped systems are characterized by high electron density and usually more number of subbands are populated [9]. Several experimental investigations have also been made to study 2DEG and also electron transport in delta-doped systems both theoretically and experimentally. [11–18]. Gonzalez *et al.* [19] have calculated the low temperature

electron mobility in  $\delta$ -doped GaAs. They have considered a multisubband system and analysed the effect of screening of the ionised impurities on subband mobility. They have shown that the Random Phase approximation proves quite good in predicting subband mobility. However, their calculations were based on the assumption of an ideal zero thickness impurity layer.

Hai *et al* [20–22] have made a study of electron transport properties of  $\delta$ -doped semiconductor systems. They have obtained the subband electronic structure of Si delta-doped GaAs by solving the coupled Schrodinger equation and the Poisson's equation selfconsistently. The screening of the ionised impurity scattering potential by 2DEG is taken into account through the static dielectric function within the Random Phase Approximation. The effect of intersubband scattering and screening on subband electron mobility has been investigated. Attempts have also taken to investigate electron mobility by increasing the number of delta-doped layers [23].

In addition to delta-doping in bulk materials, delta-doping in quantum wells (QWs) has attractive features in addition to confinement of the QWs [24–26].

### 3. Barrier delta-doped systems

A great deal of interest has also been developed in the study of the advantages of  $\delta$ -doping in the barrier region of heterostructures. Schubert *et al.* have demonstrated that in modulation doped structures the carrier mobility increases when the thickness of the doping layer is reduced and a peak value of  $\mu$  is achieved when the distribution of dopants has a  $\delta$ -function like profile [27]. In modulation doped GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$  systems, the  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  layer uniformly doped with Si contains a large number of Dx centers, which cause instability in the device operation due to persistent photoconductivity effect [28]. It has been shown that the Dx centers can be reduced by replacing the uniformly doped  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  layer by a Si – delta-doped layer [29]. Attempts have also been made to analyse the effect of barrier material on the 2DEG in delta-doped GaInAs based quantum well systems [30].

Hsu *et al* [31] have proposed a delta-doped GaAs/ $\text{In}_x\text{Ga}_{1-x}\text{As}$ /GaAs pseudomorphic structure which manifested high mobility and high 2DEG concentration. Here the GaAs barrier is delta-doped with Si. Subsequently Hsu *et al* [32] have fabricated highly strained delta- and uniformly-doped InP/InGaAs/InP high electron mobility transistor (HEMT) structure with InGaP cap layer. Attempts have also been made to improve the device performance [33,34].

A. Bebinski *et al.* [35] have studied the transport and quantum electron mobility in modulation Si delta-doped pseudomorphic GaAs/InGaAs/AlGaAs quantum wells. Huang *et al* [36] have investigated the subband properties of Si  $\delta$ -doped pseudomorphic InGaAs/GaAs heterostructures by solving the Schrodinger-Kohn-Sham equation and the Poissons equation selfconsistently. They have considered different  $\delta$ -doping configurations by placing the  $\delta$ -doping layer at various positions with respect to InGaAs well.

### 4. Double quantum well systems

A double quantum well (DQW) structure provides an ideal system for studying tunneling dynamics [37]. The modulation doping of a coupled DQW creates two parallel 2DEG

layers which exhibits interesting properties [38,39]. When two wells are brought closer, mixing of subband wave functions occurs leading to splitting in the subband energy levels. By changing the material parameters such as well width, barrier width, doping concentration, *etc.* the subband energy levels can be varied and hence it is possible to obtain a system in which more than one subband can be occupied. Several attempts have been made to study the electronic structure, optical and transport properties of double quantum well structures [40–46].

Recently Sahu and Patnaik have studied the low temperature electron mobility  $\mu_n$  in barrier delta-doped GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As double quantum well structures by considering ionised impurity scattering [43–44]. We have also considered GaAs/In<sub>x</sub>Ga<sub>1-x</sub>As double quantum well systems [45]. Unlike the GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As systems, here the well region is the alloy layer (InGaAs). Hence one has to consider the effect of alloy disorder scattering in addition to the ionised impurity scattering. We analyse the effects of screening on the alloy disorder potential, which has been normally neglected due to its short range nature. It has been showed that the effect of intersubband coupling and screening of the scattering potentials are important in studying  $\mu_n$  in a multisubband system.

## 5. Subband electron mobility

We shall discuss low temperature electron mobility  $\mu_n$  limited by ionized impurity and alloy disorder scatterings [7]. The alloy disorder scattering is of short range in order and normally the effect of screening is neglected [47]. However, it is interesting to demonstrate the effects of screening and intersubband coupling on the alloy disorder scattering [45].

Using linear response theory [1] the screened potential  $V^{\text{eff}}(q)$  and the external impurity potential  $V(q)$  can be related through the inverse dielectric matrix  $\epsilon_{nm,n'm'}^{-1}$ . Within random phase approximation (RPA) the dielectric matrix can be written as

$$\epsilon_{nm,n'm'}(q) = \delta_{nm}\delta_{nm'} + (q_s/q)F_{nm,n'm'}(q)\chi_{n'm'}^0(q), \quad (1)$$

where  $q_s = 2me^2/\epsilon_0\hbar^2$ ,  $F$  is the Coulomb form factor and  $\chi$  is the static electron density-density correlation function without electron-electron interaction [1].

For a multisubband system the expressions for the subband transport life time  $\tau_n$  satisfies a coupled linear equation and contains the intra-subband and inter-subband scattering processes in a mixed way [28,43–45]. For a single occupied band ( $n = 0$ ) one can write

$$\frac{1}{\tau_0} = B_{00}. \quad (2)$$

For double occupied bands ( $n = 0,1$ )

$$\frac{1}{\tau_0} = \frac{(B_{00} + C_{01})(B_{11} + C_{10}) - D_{01}D_{10}}{(B_{11} + C_{10}) + (E_{F1}/E_{F0})^{1/2} D_{01}} \quad (3)$$

$$\frac{1}{\tau_1} = \frac{(B_{00} + C_{01})(B_{11} + C_{10}) - D_{01}D_{10}}{(B_{00} + C_{01}) + (E_{F0}/E_{F1})^{1/2} D_{10}} \quad (4)$$

The scattering rates  $B_{nn}$ ,  $C_{nm}$  and  $D_{nm}$  are expressed in terms of the scattering matrix elements  $V_{nm}(q)$  as

$$\begin{aligned} B_{nn} &= m/(\pi \hbar^3) \int_0^\pi d\theta (1 - \cos\theta) |V_{nn}^{\text{eff}}(q_{nn})|^2, \\ C_{nm} &= m/(\pi \hbar^3) \int_0^\pi d\theta |V_{nm}^{\text{eff}}(q_{nm})|^2, \\ D_{nm} &= m/(\pi \hbar^3) \int_0^\pi d\theta \cos\theta |V_{nm}^{\text{eff}}(q_{nm})|^2 \end{aligned} \quad (5)$$

$C_{nm} = C_{mn}$  and  $D_{nm} = D_{mn}$ . Here  $k_{Fn} = (2mE_{Fn}/\hbar^2)^{1/2}$  and  $q_{nl} = [k_{Fn}^2 + k_{Fl}^2 - 2k_{Fn}k_{Fl}\cos\theta]^{1/2}$ .  
 $E_{Fn} = E_F - E_n$ .

The screened ionized impurity potential and alloy disorder potentials are given by

$$|V_{nm}^{\text{imp}}(q)|^2 = \frac{4\pi^2 e^4 N_0}{\epsilon_0^2 q^2} \left[ \int_{-d/2}^{d/2} dz_l \left| \sum_{n'm'} \epsilon_{nm,n'm'}^{-1}(q) P_{n'm'}(q, z_l) \right|^2 \right], \quad (6)$$

$$P_{n'm'}(q, z_l) = \int_{-\infty}^{\infty} dz \psi_n(z) \psi_{m'}(z) e^{-iq|z - z_l|}. \quad (7)$$

$$\begin{aligned} |V_{nm}^{\text{AL}}(q)|^2 &= [a^3 (\delta V)^2 x(1-x)/4] \\ &\times \left[ \int_{-w_b/2}^{-w_b/2 + w_w} dz \left| \sum_{nm} \psi_n(z) \psi_m(z) \epsilon_{nm,n'm'}^{-1}(q) \right|^2 + \int_{w_b/2}^{w_b/2 + w_w} dz \left| \sum_{nm} \psi_n(z) \psi_m(z) \epsilon_{nm,n'm'}^{-1}(q) \right|^2 \right], \end{aligned} \quad (8)$$

where  $a$  is the lattice constant of the alloy layer,  $\delta V$  is the alloy scattering potential and  $x$  is the alloy fraction. The indices  $n'$  and  $m'$  run over all the subband levels. However, one has to limit the sum such as considering the filled levels.

We have considered a double quantum well structure (symmetric about  $z = 0$ ). The middle barrier is of width  $w_b$ . The adjacent wells are of width  $w_w$ . One can keep the  $\delta$ -doped layer at the centre of the structure (within the middle barrier) [43] or can place it in the side barriers [44,45] keeping the middle barrier undoped. The width of the Si delta-doped layer is kept constant  $d$  Å. The doping concentration is  $N_0 \text{ cm}^{-3}$ . The electrons are transferred to the adjacent wells. Band bending occurs due to the Coulomb interaction and the alignment of the Fermi level ( $E_F$ ) throughout the system. This gives rise to symmetric triangular-like potential wells within the well layers. When the delta-doped layer lies within the central barrier, the triangular-like potential wells lie near the interfaces with the central barrier. Whereas, the delta-doping layer being in the side barriers, the triangular-like potential wells occur near the interfaces with the side

barriers. In any case, the electrons are confined to two narrow strips along the interface planes ( $xy$ -plane) forming two sheets of 2DEG [1] coupled through a thin central barrier, leading to quantization of the energy bands into subband structures along the growth direction ( $z$ -axis).

The confinement potential  $V(z)$ , the subband energy eigen values  $E_n$  and wave functions  $\psi_n(z)$  need to be obtained numerically from a selfconsistent solution of the coupled one-dimensional Schrodinger equation and the Poisson's equation. However, for sake of simplicity one can obtain an analytical expression for  $V(z)$  by solving the Poisson's equation by adopting variational trial wave function approach [1]. The bound subband energy levels  $E_n$  and wave functions  $\psi_n(z)$  are numerically obtained by using multistep potential method [48].

## 6. Discussion

The subband mobility of double quantum well systems have been elaborately studied by changing the well width, barrier width, doping concentration, etc. [43–45]. In the present invited paper we shall point out some salient features such as the effects of screening and intersubband coupling on the subband electron mobility.

From the symmetry of the double quantum well structures and the properties of the form factor [43–45], we note that the intersubband terms  $C_{01}$  and  $D_{01}$  (eq. 5) remain almost unchanged compared to the unscreened values. Screening causes reduction in  $B_{nn}$  and hence enhances the subband mobility. Therefore the effect of screening is felt through the intrasubband scattering rate matrix elements  $B_{00}$  and  $B_{11}$  only.

We shall now discuss the important aspects relating to intersubband coupling and screening of the scattering potentials on electron mobility  $\mu_n$ . From eqs. 3 and 4 we see that for a system with two-subband occupancy, the subband transport rate  $1/\tau_0$  and  $1/\tau_1$  contains the intrasubband ( $B_{00}$  and  $B_{11}$ ) and intersubband ( $C_{01}$  and  $D_{01}$ ) terms in a mixed way, unlike that of a single subband system which contains a single term (eq. 2). Thus intersubband scattering affects the mobility through these additional terms. In addition, intersubband coupling also affects the intrasubband terms  $B_{00}$  and  $B_{11}$  through the dielectric matrix element occurring in the screened scattering potential  $V^{\text{eff}}(q)$  eq. (1) thus leading to change in mobility.

Another important feature of intersubband scattering is the occurrence of sudden drop in subband mobility at the onset of occupation of a higher subband. We note that when the Fermi level is just above the second subband, eq. (3) reduces to

$$1 = B_{00} + C_{01}.$$

Comparing the above equation with eq. (2), one can find that the drop in  $\mu_0$  is due to the contribution of additional intersubband terms  $C_{01}$ . However, we note that at the onset of occupation of the second subband there is a discontinuity in the dielectric function matrix which also causes a sudden change in the intrasubband scattering rate  $B_{00}$ . Therefore the change in  $\mu_0$  is due to the combination of both the effects. We have shown. [43,45] that for impurity scattering the drop in  $\mu_0$  is mostly due to the sudden

change in  $B_{00}$ . Whereas, for alloy scattering there is no discontinuity in  $B_{nn}$ . The drop in  $\mu_0$  mostly due to the additional contributions of intersubband term  $C_{01}$ .

## 7. Conclusion

In this invited talk we briefly review the work done on low temperature electron mobility in various  $\delta$ -doped semiconductor structures. We consider a double quantum well structures, which exhibits the tunneling coupling in addition to confinement effect. We have shown that in a multi subband system the subband electron mobility contains the intrasubband and intersubband scattering processes in a mixed way. We discuss the importance of intersubband coupling on electron mobility. The effect of screening of the scattering potentials by 2DEG on subband electron mobility is also discussed. The results can be utilized for device applications.

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